# RESEARCH AND TECHNICAL NOTES

# DIFFRACTION BY HALF SCREEN WITH FINITE ACOUSTIC IMPEDANCE——A MODIFICATION TO AN EMPIRICAL FORMULA*

WANG SHUOZHONG (王朔中)

(*Institute of Acoustics, Academia Sinica*)
Received July 27, 1983

ABSTRACT

A modification to an existing empirical solution of diffraction by a half screen with a finite acoustic impedance is proposed, giving the formula a more balanced form with better accuracy when the screen is not opaque. This is obtained by introducing the transmission coefficient into the existing expression which only includes the reflection coefficient. The improvement is verified by experiments.

## I. INTRODUCTION

The search for analytic solutions to problems of diffraction involving a finite acoustic impedance is uusally difficult. Rigorous approaches lead to results in complicated mathematical forms, which are often inconvenient in practical applications. On the other hand, some empirical formulae, based upon analyses of the solutions to problems under idealized conditions, have been suggested[1-6]. The idealized condition here means that the objects are made of either absolutely soft or absolutely rigid materials. It is clear that these formulae have the drawback of lacking theoretical rigour, due to their intuitive nature. Nevertheless, they have received a good deal of attention from practical workers because of their simplicity and reasonable accuracy in many circumstances[2, 3, 7].

This paper proposes a modification to an existing empirical formula of diffraction by a semi-infinite screen. Compared with the previous formula, the new formula has a more balanced form and better accuracy.

## II. MODIFICATION TO THE FORMULA OF DIFFRACTION BY A HALF SCREEN

A commonly adopted empirical method is to introduce the reflection coefficient of the diffracting object into a solution of diffraction by an object having ideal surface conditions. Consider the following solution to the problem of diffraction by an ideal half screen,

* Chinese text published in *Acta Acustica*, 9 (1984), No. 5, 333.

assuming that the incident wave is a plane wave with a unity amplitude, propagating in a direction normal to the edge of the screen[8, 9].

$$p_d(r,\ \theta)=\frac{-\exp[-j(kr+\pi/4)]}{\sqrt{8\pi kr}}\left[\frac{F\{2kr\cos^2[(\theta-\theta_0)/2]\}}{\cos[(\theta-\theta_0)/2]}\mp\frac{F\{2kr\cos^2[(\theta+\theta_0)/2]\}}{\cos[(\theta+\theta_0)/2]}\right] \quad (1)$$

The polar coordinate system is defined in Fig. 1. In the equation, the signs correspond to absolutely soft (−) and absolutely rigid (+) boundary conditions respectively. The incident direction is represented by $\theta_0$. $F(x)$ is a modified Fresnel integral:

$$F(x)=2j\sqrt{x}\ e^{jx}\int_{\sqrt{x}}^{\infty} e^{-jt^2}dt \quad (2)$$

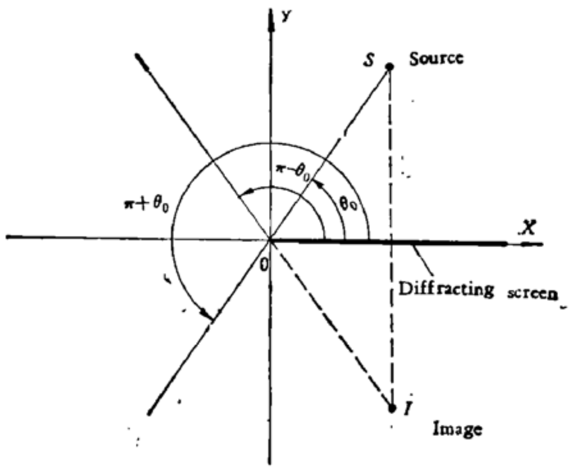



Fig. 1 Diffraction by a semi-infinite screen.

It is seen that this solution is composed of two terms of an identical function with different arguments $(\theta-\theta_0)$ and $(\theta+\theta_0)$. Therefore it can be written as

$$p_d(r,\ \theta)=f(r,\ \theta-\theta_0)\mp f(r,\ \theta+\theta_0) \quad (3)$$

As a function of $\phi$, $|f(r,\ \phi)|$ has a sharp maximum at $\phi=\pi$. Its value falls fairly steeply as $\phi$ departs from $\pi$ (Fig. 2). Consequently, the first term in Eq. (3) is dominant in

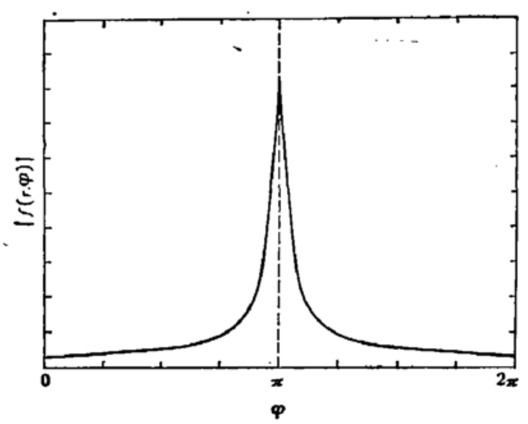



Fig. 2 $|f(r,\phi)|$ as a function of $\phi$, where $r$ is fixed.

the region close to the shadow boundary ($\theta-\theta_0=\pi$), while the second term is dominant in the region close to the reflection boundary ($\theta+\theta_0=\pi$). Therefore, these two terms are attributed respectively to the sound source and its image (see Fig. 1).

When the screen has a finite acoustic impedance, the image strength is reduced by a factor of $R(\theta_0)$. $R(\theta_0)$ is the reflection coefficient of the screen, which is a function of the material's acoustic property and the angle $\theta_0$. Thus the term of image contribution in Eq. (3) should be multiplied by $R(\theta_0)$:

$$p_d(r,\ \theta)=f(r,\ \theta-\theta_0)+R(\theta_0)f(r,\ \theta+\theta_0) \tag{4}$$

When the screen is ideally soft or rigid, $R=\mp 1$, and Eq. (3) is retrieved.

This intuitive treatment is vulnerable to criticism because it violates the principle of reciprocity, since $R(\theta_0)$ is not reciprocal. In spite of this, the formula has been proven useful in some practical situations[2, 3]. Also it is interesting to note that Williams[10] has found his results of the rigorous solution showing close agreement with the results of Raman *et al.*[1] The Raman solution is in the form of Eq. (4).

The finite impedance of the material affects the field in two aspects. The first is the reduction of reflected wave and the accompanying phase shift; and the second is the penetration of wave energy through the screen. Imagine that the effect of energy penetration increases gradually, hence the reflection decreases. In the extreme, the screen becomes completely transparent, so that $R(\theta_0)=0$. In accordance with Eq. (4), only the first term is left:

$$p_d(r,\ \theta)=f(r,\ \theta-\theta_0) \tag{5}$$

This is in fact the diffracted field of an absorbent screen. The conclusion, however, is illogical since a so-called "transparent screen" is virtually nothing, and a nonexistent object cannot produce any diffracted waves. In other words, the incident wave should remain undisturbed in this case.

To make the formula suitable for screens which are not completely opaque, the transmission coefficient $T(\theta_0)$ should be included in the formula. Thus Eq. (4) is modified as

$$p_d(r,\ \theta)=[1-T(\theta_0)]f(r,\ \theta-\theta_0)+R(\theta_0)f(r,\ \theta+\theta_0), \tag{6}$$

where $T(\theta_0)$ and $R(\theta_0)$ can be calculated from the equations given in ref. [11]. They can also be obtained experimentally.

When the screen is nearly opaque so that $|T(\theta_0)|\ll 1$, Eq. (4) is obtained. For the extreme case of a transparent screen, $T(\theta_0)=1$, and $R(\theta_0)=0$. In this case no diffracted wave exists. It is therefore clear that Eq. (6) is in a more balanced form than the previous formula. The new expression also gives better accuracy as is verified by experiments which are described in the following section.

## III. Experiments

The purpose of the experiments was to measure the acoustic pressure distribution behind a screen submerged in water and insonified by a plane wave. The geometry is shown in Fig. 3. The screen used was a steel plate with a thickness of 0.59 mm. It was big enough to be treated as semi-infinite. The direction of incidence was perpendicular to the screen and the wavelength in water was 30 mm. Under these conditions, the reflection and transmission coefficients can be worked out:

$$R=0.628-0.482j, \qquad T=0.372+0.482j$$

The acoustic field was scanned along a line $PQR$, at a distance $y_0$ behind the screen with a small hydrophone whose dimension was much less than the wavelength.

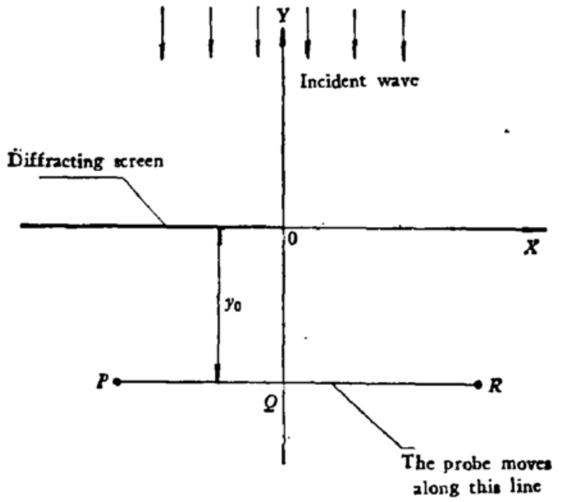



Fig. 3 Experimental arrangement.

The experimental results are shown in Fig. 4, in which the "+" signs represent the measured pressure amplitude. The two curves in each plot show the results computed from the previous formula Eq. (4) (dashed) and the proposed new formula Eq. (6) (solid) respectively. The form of function $f(r, \phi)$ is derived from Eq. (1). The curves in Fig. 4 represent the total pressure, that is to say,

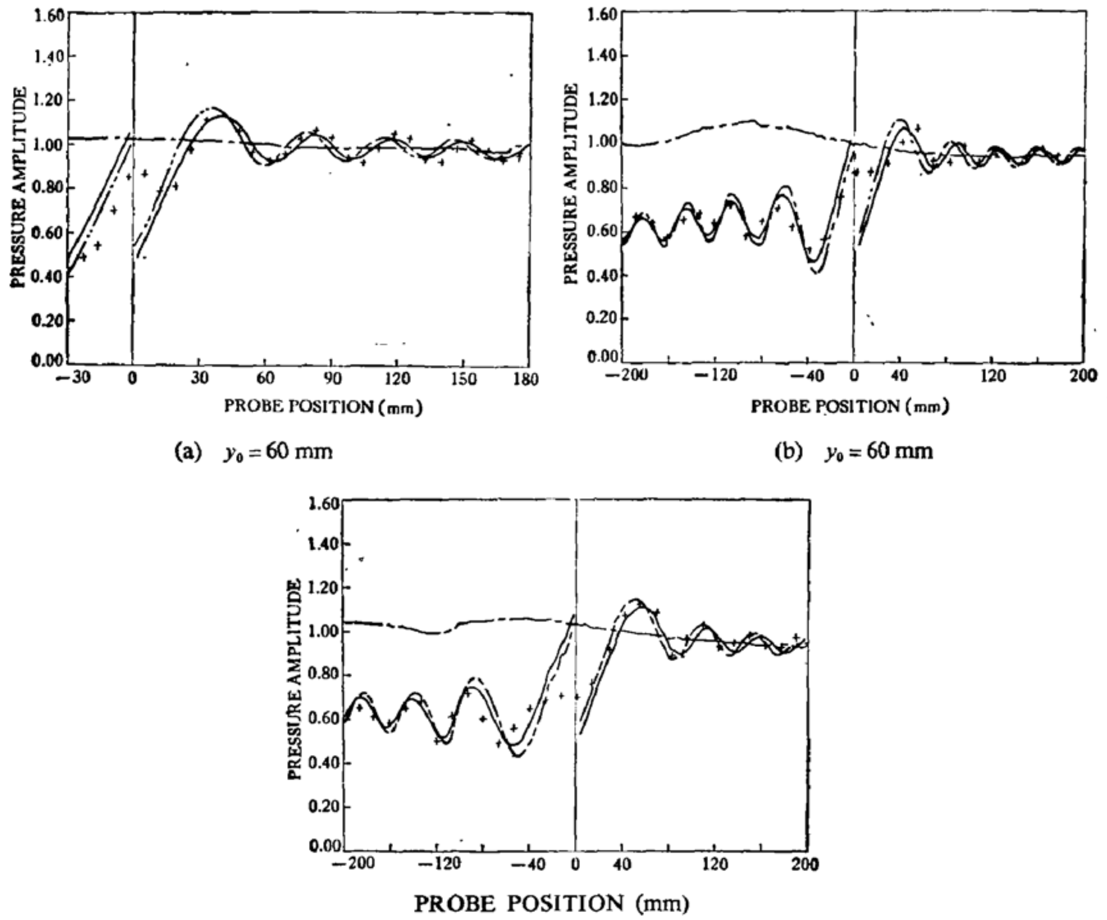
$$p=\begin{cases} p_i+p_d & (x>0), \\ p_t+p_d=Tp_i+p_d & (x<0), \end{cases} \tag{7}$$

where $p_i$, $p_t$ and $p_d$ denote incident, transmitted and diffracted waves respectively. In the computation, the measured values of $p_i$ were used. These measurements were taken along the same path $PQR$ shown in Fig. 3 in the absence of the diffracting screen. In evaluating $p_d$, the pressure values measured at the origin of the coordinate system when the screen was removed were used as the incident pressure $p_{i0}$. All the pressure values in the plots have been normalized with respect to $|p_{i0}|$.

It is obvious from Fig. 4 that the accuracy of the empirical formula is improved by the introduction of $T(\theta_0)$. Table 1 gives the root mean square errors calculated from Fig. 4. The RMS errors is defined by

$$E=\sqrt{\frac{1}{N}\sum(p_{\text{computed}}-p_{\text{measured}})^2} \tag{8}$$

where the summation is taken over all measured points outside the region $|x|<\lambda$, and $N$ is the number of points.




(a) $y_0 = 60$ mm

(b) $y_0 = 60$ mm

(c) $y_0 = 120$ mm

Fig. 4 Pressure amplitude distribution behind a semi-infinite plate.

**Table 1**
RMS errors calculated from Fig. 4

| Equation used | Figure 4 | | |
|---|---|---|---|
| | (a) | (b) | (c) |
| Eq. (4), where $T(\theta_0)$ is not included | 0.042 | 0.080 | 0.071 |
| Eq. (6), where $T(\theta_0)$ is included | 0.021 | 0.059 | 0.052 |

The high level of acoustic pressure in the geometrical shadow region is obviously due to wave penetration through the screen. The oscillation of the amplitude is the result of interference between the transmitted and diffracted waves.

The curves based on both Eq. (4) and Eq. (6) are discontinuous at $x = 0$. This exhibits yet another drawback of the empirical approach. The introduction of $R(\theta_0)$ and $T(\theta_0)$ in the expression makes the formula invalid in the vicinity of the geometrical boundaries. This is the reason why the region close to the shadow boundary ( $|x| < \lambda$) is excluded in calculating RMS errors.

## IV. Discussion

The empirical method gives useful solutions to some practical problems involving a finite acoustic impedance, despite its nonrigorous nature and drawbacks such as violation of reciprocity and discontinuities in certain regions. The proposed introduction of a transmission coefficient into the formula improves its accuracy when the diffracting screen is penetrable to the incident wave. If the transmission effect is negligible, the modified formula reduces to its original form.

In calculating the reflection and transmission coefficients, the equations in Sect. 1.5 of ref. [11] have been employed, where shear waves are ignored. If shear waves were taken into account, the results could have been more accurate.

## References

[1] Raman, C.V. and Krishnan, K.S., "The Diffraction of Light by Metallic Screens", *Proc. Roy. Soc.*, **A116** (1927), 254—267.

[2] Jonasson, H.G., "Diffraction by Wedges of Finite Acoustic Impedance with Applications to Depressed Roads", *J. Sound Vib.*, **25** (4) (1972), 557—585.

[3] Nakli, N., "Modifying the Directional Patterns of Transducer Arrays by Near-Field Perturbation", Ph. D. Thesis (University of Birmingham, England, 1978).

[4] Butler,G.F., "A Note on Improving the Attenuation Given by a Noise Barrier", *J. Sound Vib.*, **32** (1974), 376—379.

[5] Rawlins, A.D., "Diffraction by an Absorbing Wedge", *J. Sound Vib.*, **41** (3) (1975), 391—393.

[6] Butler, G.F., "Diffraction by a Wedge with an Impedance Condition at the Surface", *J. Sound Vib.*, **47** (2) (1976), 283—286.

[7] Wang Shuozhong (Wang. S.), "An Application of Diffraction Theory to Echo Formation", *Chinese J. Acoust.*, **2** (1983), No. 4, 273—292.

[8] Wang, S., "The Diffracted Acoustic Field around a Wedge", *Chinese J. Acoust.*, **3** (1984), No. 2, 121—132.

[9] Pathak, P.H. and Kouyoumjian, R.G., "The Dyadic Diffraction Coefficient for a Perfectly Conducting Wedge" (Elctroscience Lab. Report 2183-4, Ohio University, 1970).

[10] Williams, W.E., "Diffraction of an E-Polarized Plane Wave by an Imperfectly Conducting Wedge", *Proc. Roy. Soc.*, **A252** (1959), 376—393.

[11] Brekhovskikh, L., "*Waves in Layered Media*", Sects. 1.5 and 1.6, Academic Press, 1960.