Modification to first-order Born approximation for improved prediction of scattered sound from weakly scattering objects*

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Abstract When deriving the Fourier diffraction theorem based on the first-order Born approximation, the difference between wave number of the scattering object and that of the surrounding medium is ignored, causing substantial errors in sound scattering prediction. This paper modifies the Born approximation by taking into account the amplitude and phase changes between the scattering object and the water due to the wave number difference. By changing the radius and center position of the sampling circle in the Fourier domain, accuracy of the predicted sound scattering is improved. With the modified Born approximation, the computed far-field directional pattern of the scattered sound from a circular cylinder is in good agreement with the rigorous solution. Numerical calculations for several objects with different shapes are used to show applicability and effectiveness of the proposed method.

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Abstract: When deriving the Fourier diffraction theorem based on the first-order Born approximation, the difference between wave number of the scattering object and that of the surrounding medium is ignored, causing substantial errors in sound scattering prediction. This paper modifies the Born approximation by taking into account the amplitude and phase changes at the interface between the scattering object and the water due to the wave number difference. By changing the radius and center position of the sampling circle in the Fourier domain, accuracy of the predicted sound scattering is improved. With the modified Born approximation, the computed far-field directional pattern of the scattered sound from a circular cylinder is in good agreement with the rigorous solution. Numerical calculations for several objects with different shapes are used to show applicability and effectiveness of the proposed method.

Introduction

Analytical and numerical methods can be used to solve scattering problems that involve scattering of sound waves from objects made of different materials [1, 2, 3]. Analytical solutions are only available in some ideal cases, while most numerical approaches are based on grid division of the computation domain, implying a heavy computational burden. To meet the practical requirements, it is necessary to develop methods for fast prediction of the characteristics of scattering fields.

Acoustic diffraction tomography, *i.e.*, reconstruction of the cross-section of a scattering object from the measurements of scattered waves, is a typical inverse problem, which can be solved by using the Fourier diffraction theorem (FDT) [4]. It has been shown that the far-field scattered sound pressure measured along a closed curve around the object is proportional to the values sampled on a circle in the 2D Fourier transform of the object. The circle's radius is the incident wave number k_0, and its center is located at k_0 from the origin [5]. Reversed application of the Fourier diffraction theorem can treat a forward problem, viz., predicting scattered fields from the incidence parameters, and the known object geometry and acoustical properties. The second author of this paper established an image model based on the shape and acoustical parameters of the object and its surrounding medium, and calculated inverse DFT of samples taken on a pair of semi-circles in the Fourier domain to give generalized projection of the scattered sound in the space. That provides a means of fast

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prediction of scattered wave characteristics [6]. Compared with the grid-and-iteration based methods such as FDTD, this method reduces the computation complexity by at least two orders of magnitude, and is therefore potentially applicable to practical engineering.

In the above mentioned method, the first-order Born approximation was used in deriving the Fourier diffraction theorem, assuming the same wave number of the incident sound wave in the water and inside the scattering object. That inevitably causes errors since the wave numbers are actually different inside and outside the object.

In calculating the forward- and backward-scattering from a spherical target by integration, Saha *et al.* assume that the sound field inside the target approximately equals the incident wave, but with a modified wave number according to the sound speed of the target material [7]. This way, the obtained forward scattered results are closer to the rigorous solution as compared to the first-order approximation. Oboznenko *et al.* use a similar method to modify the first Born approximation [8]. They predict the scattered sound waves from layered and elastic objects under weak scattering assumption, and carry out experimental and numerical studies on the effects of changing acoustic impedance of the targets on scattering. Jones *et al.* calculate the back-scattering by taking into account the different acoustic properties of the scattering object and the surrounding medium, and study the effects of the object on the scattered field's phase variations [9]. They introduce the distorted wave Born approximation (DWBA) to study scattering from fluid-filled spherical and cylindrical shells, and treat squids as cylindrical shells in their numerical computation with results comparable to experiments. These methods all produce results better than that of the first-order Born approximation.

The first-order Born approximation is applicable to weak scattering, *e.g.*, sound scattering from liquid objects such as squids, jelly fish and zooplanktons in the ocean that have acoustical impedances close to the surrounding sea water. To further improve the quality of weak scattering prediction, a modification to the first-order Born approximation is proposed, and applied to the FDT-based fast prediction of scattered sound. By considering the phase and amplitude changes due to different acoustical properties between the object and water, the frequency domain sampling circle is modified with its radius changed and center shifted. When the sound speed in the object is higher than in water, the radius becomes smaller and the center shifted toward the origin, and vice versa. It will be shown that, by doing so, the modified first-order Born approximation results in batter prediction of sound scattering as compared with the other methods introduced in the literature. Numerical computations show that errors caused by ignoring different wave number inside the object are substantially reduced, and directional patterns of the scattered sound from cylindrical objects are close to the analytical solution.

1 Sound scattering and the first-order Born approximation

Consider a two dimensional sound field as shown in Figure 1, in which a plane wave is incident toward an infinitely long cylinder with an arbitrary cross-section. The direction of incidence is perpendicular to the cylinder's axis. Let the incident direction form an angle φ with the positive x axis, thus the unit vector along the incident wave is $\hat{\mathbf{r}}_0$=(cosφ, sinφ). The vector $\mathbf{r}=(x, y)$ represents the field location, and $\mathbf{r}'=(x', y')$ is on the source of the scattered waves, *i.e.*, on the scattering object. The unit vector in the scattering direction θ is $\hat{\mathbf{r}}$ =(cosθ, sinθ). Assume that densities of the scattering object and the water are ρ_1 and ρ_0, and their sound velocities c_1 and c_0, respectively.

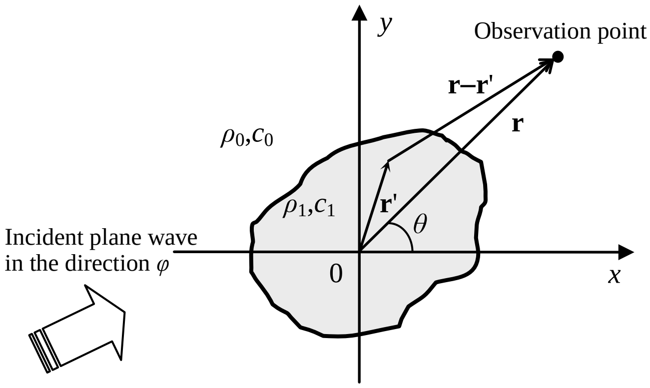


Figure 1 Two-dimensional sound scattering

The scattered sound filed satisfies the Helmholtz equation:

$$(\nabla^2 + k_0^2)\,p_s(\mathbf{r}) = -o(\mathbf{r})[\,p_i(\mathbf{r}) + p_s(\mathbf{r})] \tag{1}$$

where $p_i(\mathbf{r})$ and $p_s(\mathbf{r})$ are the incident and scattered sound pressure respectively. The spatial distribution of the refractive index $[c_0/c(\mathbf{r})-1]$ in the cross-section of the object and water can be viewed as an image $o(\mathbf{r})$, where c_0 is the sound speed in water. For a homogeneous object, $c(\mathbf{r})$ equals a constant c_1 so that $o(\mathbf{r})$ is a binary image. k_0 is the wave number in water. When $p_i(\mathbf{r})>>p_s(\mathbf{r})$, the first-order Born approximation applies, and the scattered sound pressure can be obtained by the following integration:

$$p_s(\mathbf{r}) = \int o(\mathbf{r}')\,p_i(\mathbf{r}')\,g(\mathbf{r}\,|\,\mathbf{r}')\,d\mathbf{r}' \tag{2}$$

where $p_i(\mathbf{r}')=\exp(jk_0\hat{\mathbf{r}}_0\cdot\mathbf{r}')$. The Green's function $g(\mathbf{r}|\mathbf{r}')$ is the zero-order Hankel function of the first kind [10], with the far-field asymptotic solution:

$$g(\mathbf{r}\,|\,\mathbf{r}') = \sqrt{\frac{2}{\pi|\mathbf{r}-\mathbf{r}'|}}\exp(jk_0|\mathbf{r}-\mathbf{r}'| - j\frac{\pi}{4}) = \sqrt{\frac{2}{\pi r}}\exp[jk_0(r-\hat{\mathbf{r}}\cdot\mathbf{r}') - j\frac{\pi}{4}] \tag{3}$$

where $(\hat{\mathbf{r}}\cdot\mathbf{r}') = x'\cos\theta + y'\sin\theta$. Substitute the incident wave and the Green's function into (2), and let the 2D Fourier transform of $o(\mathbf{r})$ be $O(\mathbf{k})$. The far-field scattered pressure based on the first-order Born approximation is therefore obtained:

$$p_s(r,\varphi,\theta) = \sqrt{\frac{2}{\pi r}} 4\pi^2 O\left(k_0\cos\theta - k_0\cos\varphi, k_0\sin\theta - k_0\sin\varphi\right)\exp(jk_0 r - j\frac{\pi}{4}) \tag{4}$$

From (4), the directional pattern of the far-field scattering is determined by the term $O(k_0\cos\theta - k_0\cos\varphi, k_0\sin\theta - k_0\sin\varphi)$. It is therefore observed that the scattered sound pressure is proportional to the samples taken from a circle in the 2D Fourier transform of the object image. The circle has a radius k_0 and is centered at $(-k_0\cos\varphi, -k_0\sin\varphi)$. When φ=0°, the center is at $(-k_0, 0)$. Changing the incident direction, the sampling circle is rotated around the origin. Figure 2 shows 4 sampling circles for φ=0°, 90°, 180° and 270°. If $\theta=\varphi$, the directional factor is $O(0, 0)$, meaning that the spectral domain origin corresponds to the forward scattering. Likewise, $\theta=\pi-\varphi$ means back-scattering, corresponding to the point $(-2k_0\cos\varphi, 0)$ in the 2D spectrum.

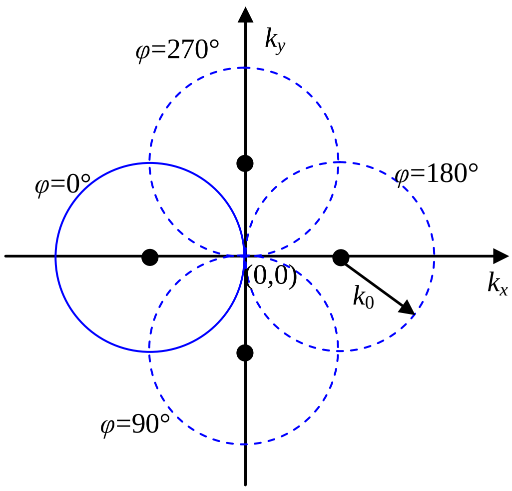


Figure 2 Sampling circles in the Fourier domain based on the first-order Born approximation

If the object is inhomogeneous, $o(\mathbf{r})$ is no longer binary. This, however, does not affect the relation between the spectral sampling locus and the scattered sound field.

2 Modification to the first-order Born approximation

Taking into account the difference in sound speed and density between the object material and water, the integral equation for the scattered sound pressure is [11]:

$$p_s(\mathbf{r}) = \int \left\{ k_0^2 \gamma_\kappa o(\mathbf{r}') p_i(\mathbf{r}') g(\mathbf{r}\,|\,\mathbf{r}') + \gamma_\rho o(\mathbf{r}') \left[\nabla p_i(\mathbf{r}') \cdot \nabla g(\mathbf{r}\,|\,\mathbf{r}')\right] \right\} dr' \tag{5}$$

where γ_κ is related to the compressibility $\kappa=1/\rho c^2$, and γ_ρ is determined by the densities of the object and water, ρ_1 and ρ_0:

$$\gamma_\kappa = \begin{cases} \dfrac{\kappa_1 - \kappa_0}{\kappa_0} & \text{inside object} \\ 0 & \text{in water} \end{cases} \tag{6}$$

$$\gamma_\rho = \begin{cases} \dfrac{\rho_1 - \rho_0}{\rho_0} & \text{inside object} \\ 0 & \text{in water} \end{cases} \tag{7}$$

In view of the phase change of the sound wave when it enters the object, Jones *et al.* propose the DWBA to calculate backward scattering form a weak scatterer. They take the wave number inside the object as k_v, equivalent to $k_1=k_0(1+\alpha)$ in the present work, instead of the wave number in the water, k_0, see Equation (5) of [9]. DWBA introduces a modification to the phase of the scattered wave based on the Born approximation without considering the amplitude. With their method, good results can be obtained when the sound speed inside the object is slightly lower than that in water. For example, Jones *et al.* consider scattering from squids with the ρc ratio with respect to water being 0.911~0.934. As the acoustical impedance of the object deviating further from that of water, errors of DWBA increase steeply. To improve accuracy of the predicted scattered sound, it is necessary to make further modification to the first-order Born approximation. Thus, the following term is introduced into the expression:

$$\psi[(\hat{\mathbf{r}}_0 \cdot \mathbf{r}'), k_1] = \exp\left[jk_1\alpha(\hat{\mathbf{r}}_0 \cdot \mathbf{r}')\right] \tag{8}$$

Multiplying $\psi(\cdot)$ with $p_i(\mathbf{r}')$ and in the rectangle coordinates, the term $\nabla p_i(\mathbf{r}')\cdot\nabla g(\mathbf{r}|\mathbf{r}')$ in (5) is modified to

$$\begin{aligned} &\nabla p_i(\mathbf{r}')\psi[(\hat{\mathbf{r}}_0 \cdot \mathbf{r}'), k_1]\cdot\nabla g(\mathbf{r}\,|\,\mathbf{r}') \\ &\quad = k_1^2(1+\alpha)(\cos\varphi, \sin\varphi)\cdot(\cos\theta, \sin\theta)\, p_i(x', y')\psi[x', y']g(x, y\,|\,x', y') \end{aligned} \tag{9}$$

where $(\cos\varphi, \sin\varphi)(\cos\theta, \sin\theta) = \cos(\theta-\varphi)$. Substitute (9) into (5),

$$p_s(x, y) = k_1^2[\gamma_\kappa + \gamma_\rho(1+\alpha)\cos(\theta-\varphi)\int\int o(x', y')\, p_i(x', y')\psi(x', y')g(x, y\,|\,x', y')dx'dy' \tag{10}$$

The double integration in (10) can be evaluate as

$$\begin{aligned} &\int\int o(x', y')\, p_i(x', y')\psi(x', y')g(x, y; x', y')dx'dy' \\ &\quad = \sqrt{\frac{2}{\pi r}}\exp(jk_0 r - j\frac{\pi}{4})4\pi^2 O\left[k_1\cos\theta - k_1(1+\alpha)\cos\varphi, k_1\sin\theta - k_1(1+\alpha)\sin\varphi\right] \end{aligned} \tag{11}$$

Thus the scattered sound pressure based on the modified first-order Born approximation is

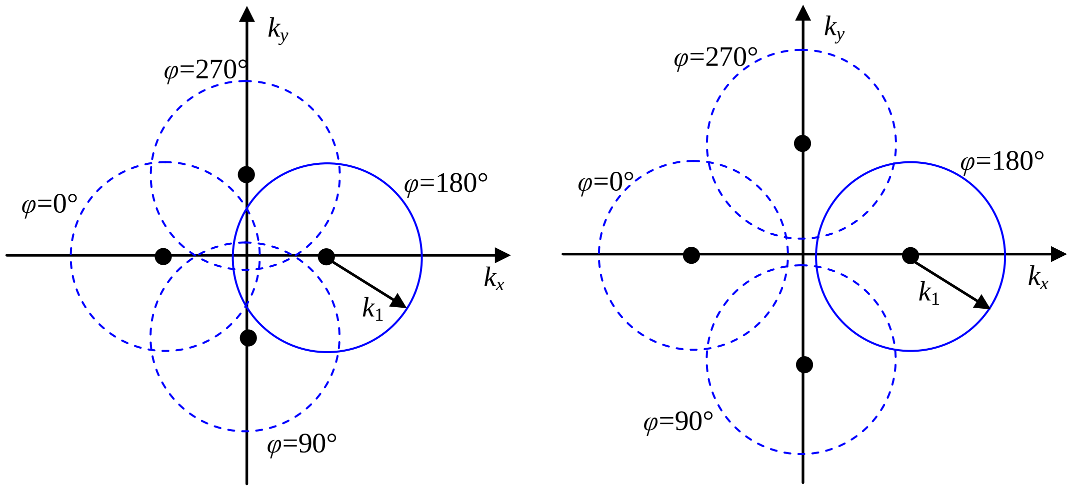
$$p_s(r, \varphi, \theta) = 4\pi^2 k_0^2(1+\alpha)^2 f(\varphi, \theta)\sqrt{\frac{2}{\pi r}}\exp(jk_0 r - j\frac{\pi}{4}) \tag{12}$$

where the directional pattern of the scattered field is

$$f(\varphi, \theta) = [\gamma_\kappa + \gamma_\rho(1+\alpha)\cos(\theta-\varphi)]O\left[k_1\cos\theta - k_1(1+\alpha)\cos\varphi, k_1\sin\theta - k_1(1+\alpha)\sin\varphi\right] \tag{13}$$

The directional pattern is related to the wave number and the parameters of the object: α, γ_κ, and γ_ρ. From the term $O[\bullet]$ in (13), the modification made lies in the changes of the sampling circle: the radius becomes k_1, and the center moves to $[-k_1(1+\alpha)\cos\varphi, -k_1(1+\alpha)\sin\varphi]$. If $\varphi=0^\circ$, the center is at $[-k_1(1+\alpha), 0]$. When the sound speed in the object is greater than that in water, *i.e.*, $\alpha<0$ and $k_1<k_0$, the radius becomes smaller, and the center moves

toward the origin. Changing the incident angle, the sampling circle rotates about the origin by the same angle. Figure 3(a) shows such a case with φ=0º, 90°, 180° and 270°. Contrarily, If $c_1 < c_0$, then $\alpha>0$ and $k_1>k_0$, the radius becomes larger, and the center moves away from the origin, as shown in Figure 3(b).



(a) Sound speed in object greater than in water (b) Sound speed in object less than water

Figure 3 Modified sampling circles

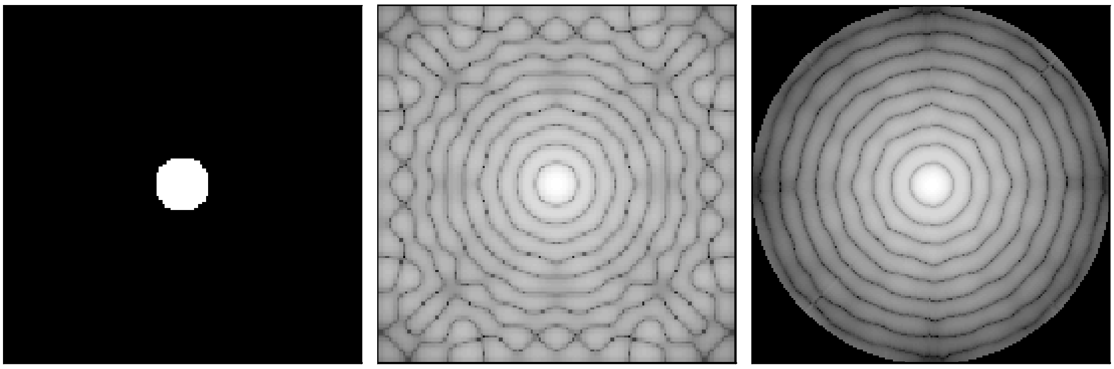
3 Numerical computation and discussion

In this section, numerical computation is carried out using the Fourier diffraction theory based on the modified first-order Born approximation. The directional pattern of the scattered sound is computed. The scattering object is infinitely long cylinders immerged in water, with the cross-section being circle, ellipse and rectangle. The incident wave propagates toward the cylinder and perpendicular to the cylinder's axis. Parameters used in the computation are listed in Table 1.

Figure 4(a) shows the cross section of a circular cylinder with radius of 1m, computation domain sized 12.8m×12.8m, and spatial sampling interval of 0.1m. According to the method described in Section 2, DFT is performed on the image representing the object and the surrounding water, and spectral samples on an appropriate circle are taken to give directional pattern of the scattering sound in the space domain. However, the sampled object appears to be jaggy due to the coarse sampling, leading to serious errors in the DFT, especially in the high frequency region, see Figure 4(b). To deal with the problem, the authors have proposed a technique based on the Radon projection and using repeated interpolation-smoothing-decimation operations to reduce the jaggedness effect and improve quality of the spectrum [12]. Shown in Figure 4(c) is the improved spectrum, from which samples are taken according to the previous section. As only values on the rectangular grid are available in the spectral domain, bilinear interpolation is performed to obtain accurate sample values on the circles.

Table 1 Geometrical and acoustical parameters used in the computation

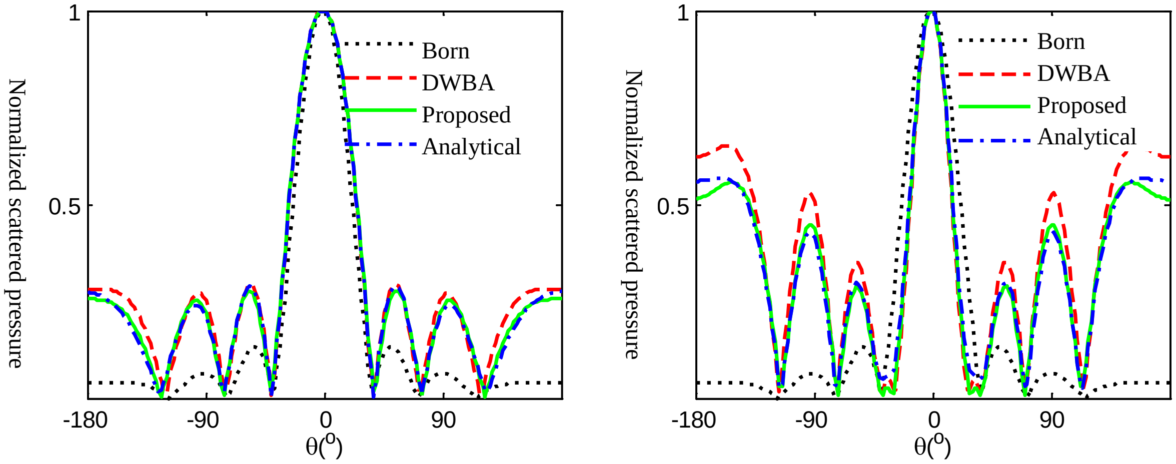
Item		Symbol	Unit	Value
Frequency		f_0	Hz	1500
Sound speed in water		c_0	m/s	1500
Density of water		ρ_0	kg/m^3	1027
Sampling interval in space		δ	m	0.1
Side of computation domain		a	m	12.8
Density of object	Material 1	ρ_1	kg/m^3	1223
	Material 2	ρ_1	kg/m^3	1350
Sound speed in object	Material 1	c_1	m/s	1549
	Material 2	c_1	m/s	1475
Size of object	Circular cylinder	$k_0R = 2\pi R/\lambda$	−	1~15
	Elliptic cylinder	k_0R_1 (R_1 is long axis)	−	10
		k_0R_2 (R_2 is short axis)	−	5
	Rectangular cylinder	k_0L_1 (L_1 is long side)	−	10
		k_0L_2 (L_2 is short side)	−	5



(a) Jaggy image in space (b) direct DFT result (c) Corrected spectrum based on [12]

Figure 4 Image model of scattering object and its 2D spectrum

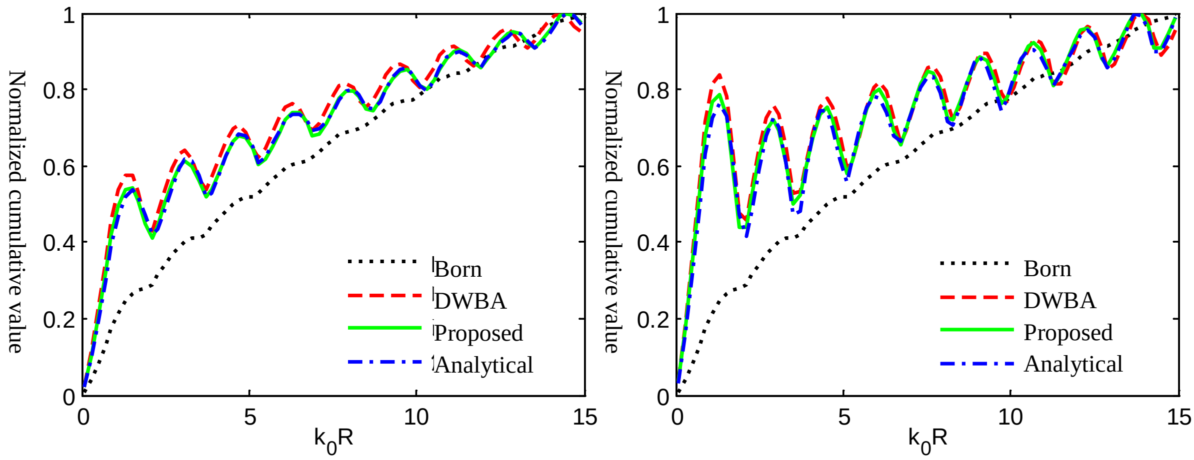
Let the incident wave propagate along the x-axis, and k_0R=6. Calculate the far-field directional pattern $f(\varphi, \theta)$ using the parameters given in Table 1. The ratio between sound speeds in the water and the object, c_0/c_1 is 0.96～1.02, and the ratio of densities 0.76~0.84. Compared to [9], $(\rho c)_0/(\rho c)_1$ is further apart from 1. In this case, errors of DWBA become large, while with the proposed method, the prediction accuracy is significantly higher, closer to the analytical solution. The computation results are shown in Figures 5(a) and 5(b), in which the analytical solution (chained blue lines), and results of the first-order Born approximation (dotted black lines), DWBA (dashed red lines), and the proposed method (solid green lines) are compared. It is observed that the original Born approximation gives the poorest results, DWBA provides improvements, and the proposed method is even better.



(a) Sound speed in object is greater (b) Sound speed in object is smaller

Figure 5 Scattering patterns obtained with different methods (k_0R=6)

Now, change the value of k_0R by using different values of the cylinder radius while keeping the incident frequency 1500Hz unchanged. Calculate the value of the far-field scattered sound pressure every 1°, and sum up over the entire range of 360°. Repeat the computation using the four methods as in Figure 5, and normalize the results with respect to the maximum of each method. The results are plotted in Figure 6, which can be used to evaluate the overall degree of closeness between the predicted scattering and the analytical solution with various k_0R values. Again, the original Born approximation produces results with large errors, and the proposed method can give results that are closest to the analytical solution.

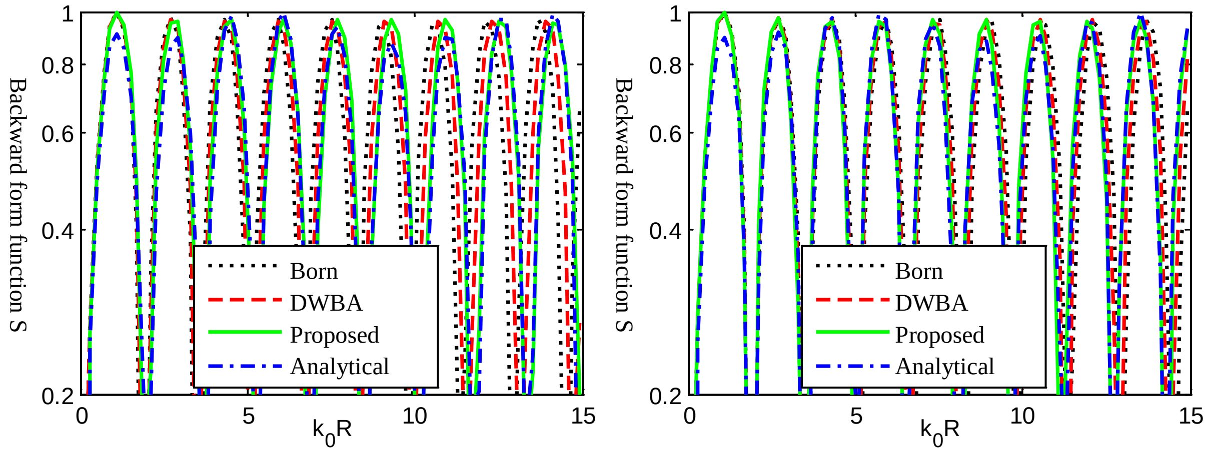


(a) Sound speed in object is greater (b) Sound speed in object is smaller

Figure 6 Normalized cumulative results of the directional characteristics over 360°

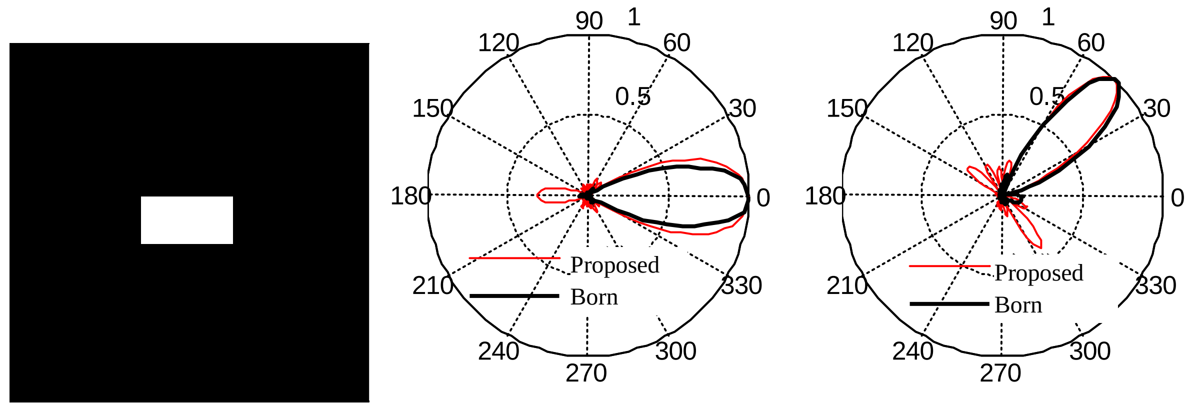
As k_0R increases, the scattered energy is progressively concentrated toward the forward direction, and the directional property becomes complicated in other directions. Assume φ=0° and θ=180°, calculate the form function $S(k_0R)$ of the normalized magnitude of the backward scattered sound pressure, and plot the results in Figure 7. The form function was used by Hickling in discussing the frequency response of scattered sound fields. Clearly, the proposed modification to the Born approximation method better results than DWBA.

Figure 8 gives computation results for objects other than circular cylinders. Assume $c_1>c_0$, and use the geometrical parameters listed in Table 1. Directional patterns with φ=0° and 45°. As expected, large lobes appear in the directions of specular reflection, which are absent in the results obtained by using the original Born approximation.

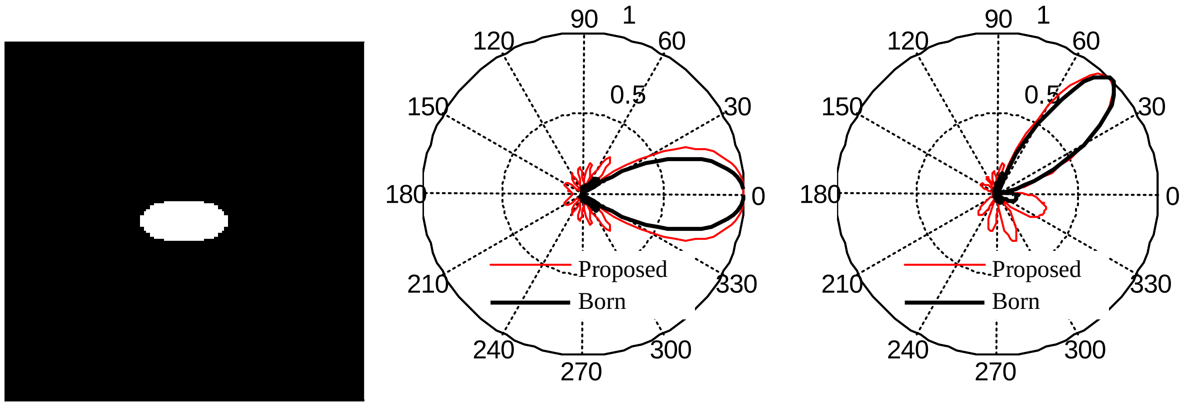


(a) Sound speed in object is greater (b) Sound speed in object is smaller

Figure 7 Function $S(k_0R)$ of backward scattered field



(a) Cross section of rectangular cylinder (b) φ=0° (c) φ=45°



(a) Cross section of elliptical cylinder (b) φ=0° (c) φ=45°

Figure 8 Directional patterns of scattered sound from objects of different shapes

4 Conclusions

Under the condition of weak scattering, the density and sound speed of the object is close to that of water so that the scattered sound field can be efficiently predicted through a reversed use of the Fourier diffraction theorem based on the first-order Born approximation. However, the first approximation ignores the influence of the wave number variation of the incident sound wave inside the object, leading to considerable errors in the scattering prediction. This paper proposes to modify the first-order Born approximation. The sampling circle in the frequency domain is shrunk or enlarged, and shifted, according to the different properties of the object. This way, the scattering prediction becomes more accurate. Numerical computations show that the proposed method outperforms the previously published DWBA.

The method proposed in the present paper has expanded the scope of application of the first-order approach. If the acoustical impedance goes further away from that of the surrounding medium, the first-order method will become inapplicable. In that case, higher-order techniques or other approaches must be sought.

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